Low momentum nucleon-nucleon potential and shell model effective interactions

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A low momentum nucleon-nucleon (NN) potential V_{low-k} is derived from meson exhange potentials by integrating out the model dependent high momentum modes of V_{NN}. The smooth and approximately unique V_{low-k} is used as input for shell model calculations instead of the usual Brueckner G matrix. Such an approach eliminates the nuclear mass dependence of the input interaction one finds in the G matrix approach, allowing the same input interaction to be used in different nuclear regions. Shell model calculations of ^{18}O, ^{134}Te and ^{135}I using the **same** input V_{low-k} have been performed. For cut-off momentum Λ in the vicinity of 2 fm^{-1}, our calculated low-lying spectra for these nuclei are in good agreement with experiments, and are weakly dependent on Λ.

21.60.Cs; 21.30.Fe; 27.80.+j

A fundamental problem in nuclear physics has been the determination of the effective nucleon-nucleon (NN) interaction used in the nuclear shell model, which has been successful in describing a variety of nuclear properties. There have been a number of successful approaches [1–4] for this determination, ranging from empirical fits of experimental data, to deriving it microscopically from the bare NN potential. Despite impressive quantitative successes, the traditional microscopic approach suffers the fate of being "model dependent" owing to the fact that there is no unique V_{NN} to start from. Moreover, as the Brueckner G matrix has traditionally been the starting point, one obtains different input interactions for nuclei in different mass regions as a result of the Pauli blocking operator.

In this work, we propose a different approach to shell model effective interactions that is motivated by the recent applications of effective field theory (EFT) and the renormalization group (RG) to low energy nuclear systems [5–8]. Our aim is to remove some of the model dependence that arises at short distances in the various V_{NN} models, and also to eliminate the mass dependence one finds in the G matrix approach, thus allowing the same interaction to be used in different nuclear regions such as those for ^{18}O and ^{134}Te. A central theme of the RG-EFT approach is that physics in the infrared region is insensitive to the details of the short distance dynamics. One can therefore have infinitely many theories that differ substantially at small distances, but still give the same low energy physics if they possess the same symmetries and the "correct" long-wavelength structure [5,8]. The fact that the various meson models for V_{NN} share the same one pion tail, but differ significantly in how they treat the shorter distance pieces illustrates this explicitly as they give the same phase shifts and deuteron binding energy. In RG language, the short distance pieces of V_{NN} are like irrelevant operators since their detailed form can not be resolved from low energy data.

Motivated by these observations, we would like to derive a low-momentum NN potential V_{low-k} by integrating out the high momentum components of different models of V_{NN} in the sense of the RG [5,8], and investigate its suitability of being used directly as a model independent effective interaction for shell model calculations. We shall use in the present work the CD-Bonn NN potential [9] for V_{NN}. In the following, we shall first describe our method for carrying out the high-momentum integration. Shell model calculations for ^{18}O, ^{134}Te and ^{135}I using V_{low-k} will then be performed. Our results will be discussed, especially about their dependence on the cut-off momentum Λ.

The first step in our approach is to integrate out the model dependent high momentum components of V_{NN}. In accordance with the general definition of a renormalization group transformation, the decimation must be such that low energy observables calculated in the full theory are exactly preserved by the effective theory. We turn to the model space methods of nuclear structure theory for guidance, as there has been much work in recent years discussing their similarity to the Wilson RG approach [4,10,11]. While the technical details differ, both approaches attempt to thin-out, or limit the degrees of freedom one must explicitly consider to describe the physics in some low energy regime. Once the relevant low energy modes are identified, all remaining modes or states are "integrated" out. Their effects are then implicitly buried inside the effective interaction in a manner that leaves the low energy observables invariant. One successful model-space reduction method is the Kuo-Lee-Ratcliff (KLR) folded diagram theory [12,13]. For the nucleon-nucleon problem in vacuum, the RG approach simply means that the low momentum T matrix and the deuteron binding energy calculated from V_{NN} must be reproduced by V_{low-k}, but with all loop integrals cut off at some Λ. Therefore, we start from the half-on-shell T-matrix

$$T(k',k,k^2) = V_{NN}(k',k) + \int_0^\infty q^2 dq V_{NN}(k',q)\frac{1}{k^2-q^2+i0^+}T(q,k,k^2). \quad (1)$$

We then define an effective low-momentum T-matrix by

$$T_{low-k}(p',p,p^2) = V_{low-k}(p',p) + \int_0^\Lambda q^2 dq V_{low-k}(p',q)\frac{1}{p^2-q^2+i0^+}T_{low-k}(q,p,p^2), \tag{2}$$

where Λ denotes a momentum space cut-off (such as Λ=$2fm^{-1}$) and $(p',p) \leq \Lambda$. We require the above T-matrices satisfying the condition

$$T(p',p,p^2) = T_{low-k}(p',p,p^2);\ (p',p) \leq \Lambda. \tag{3}$$

The above equations define the effective low momentum interaction V_{low-k}. In the following, let us show that the above equations are satisfied by the solution

$$V_{low-k} = \hat{Q} - \hat{Q}'\int\hat{Q} + \hat{Q}'\int\hat{Q}\int\hat{Q} - \hat{Q}'\int\hat{Q}\int\hat{Q}\int\hat{Q} + \ \ldots\ , \tag{4}$$

which is just the KLR folded-diagram effective interaction [12,13]. A preliminary account of this result has been reported as a work in progress at a recent conference [14].

In time dependent formulation, the T-matrix of Eq.(1) can be written as $\langle k' \mid VU(0,-\infty) \mid k\rangle$, U being the time evolution operator. In this way we can readily perform a diagrammatic analysis of the T-matrix. A general term of it may be written as $\langle k' \mid (V + V\frac{1}{e(k)}V + V\frac{1}{e(k)}V\frac{1}{e(k)}V + \cdots) \mid k\rangle$ where $e(k) \equiv (k^2 - H_o)$, H_o being the unperrurbed Hamiltonian. Note that the intermediate states (represented by 1 in the numerator) cover the entire space, and $1 = P+Q$ where P denotes the model space (momentum $\leq \Lambda$) and Q its complement. Expanding it out in terms of P and Q, a typical term of T is of the form $V\frac{Q}{e}V\frac{Q}{e}V\frac{P}{e}V\frac{Q}{e}V\frac{P}{e}V$. Let us define a $\hat{Q}$-box as $\hat{Q} = V+V\frac{Q}{e}V+V\frac{Q}{e}V\frac{Q}{e}V+\cdots$, where all intermediate states belong to Q. One readily sees that the T-matrix can be regrouped as a $\hat{Q}$-box series, namely $\langle p' \mid T \mid p\rangle = \langle p' \mid [\hat{Q} + \hat{Q}\frac{P}{e}\hat{Q} + \hat{Q}\frac{P}{e}\hat{Q}\frac{P}{e}\hat{Q} + \cdots] \mid p\rangle$. Note that all the $\hat{Q}$-boxes have the same energy variable, namely p^2.

This regrouping is depicted in Fig. 1, where each $\hat{Q}$-box is denoted by a circle and the solid line represents the propagator $\frac{P}{e}$. The diagrams A, B and C are respectively the one- and two- and three-$\hat{Q}$-box terms of T, and clearly T=A+B+C+$\cdots$. Note the dashed vertical line is not a propagator; it is just a "ghost" line to indicate the external indices. We now perform a folded-diagram factorization for the T-matrix, following closely the KLR folded-diagram method [12,13]. Diagram B of Fig. 1 is factorized into the product of two parts (see B1) where the time integrations of the two parts are independent from each other, each integrating from $-\infty$ to 0. In this way we have introduced a time-incorrect contribution which must be corrected. In other words B is not equal to B1, rather it is equal to B1 plus the folded-diagram correction B2. Note that the integral sign represents a generalized folding [12,13].

Similarly we factorize the three-$\hat{Q}$-box term C as shown in the third line of Fig. 1. Higher-order $\hat{Q}$-box terms are also factorized following the same folded-diagram procedure. Let us now collecting terms in the figure in a "slanted" way. The sum of terms A1, B2, C3... is just the low-momentum effective interaction of Eq.(4). (Note that the leading $\hat{Q}$-box of any folded term must be at least second order in V_{NN}, and hence it is denoted as $\hat{Q}'$-box which equals to $\hat{Q}$-box with terms first-order in V_{NN} subtracted.) The sum B1, C2, D3.... is $V_{low-k}\frac{P}{e}\hat{Q}$. Similarly the sum C1+D2+E3+$\cdots$ is just $V_{low-k}\frac{P}{e}\hat{Q}\frac{P}{e}\hat{Q}$. (Note diagrams D1, D2, $\cdots$, E1, E2, $\cdots$ are not shown in the figure.) Continuing this way, it is easy to see that Eqs. (1) to (3) are satisfied by the low momentum effective interaction of Eq.(4).

The effective interaction of Eq.(4) can be calculated using iteration methods. A number of such iteration methods have been developed; the Krenciglowa-Kuo [15] and the Lee-Suzuki iteration methods [16] are two examples. These methods were formulated primarily for the case of degenerate PH_0P, H_0 being the unperturbed Hamiltonian. For our present two-nucleon problem, PH_0P is obviously non-degenerate. Non-degenerate iteration methods [17] are more complicated. However, a recent iteration method developed by Andreozzi [18] is particularly efficient for the non-degenerate case. This method shall be referred to as the Andreozzi-Lee-Suzuki (ALS) iteration method, and has been employed in the present work.

We have carried out numerical checks to ensure that certain low-energy physics of V_{NN} are indeed preserved by V_{low-k}. We first check the deuteron binding energy BE_d. We have calculated BE_d using V_{low-k} for many values of Λ, and for all cases the BE_d given by V_{low-k} agrees very accurately (to 4 places after the decimal) with that given by V_{NN}. (Note that when Λ approaches ∞ V_{low-k} is the same as V_{NN}.) In Fig. 2, we present some $^1S_0$ and $^3P_0$ phase shifts calculated from the CD-Bonn V_{NN} (dotted line) and the V_{low-k} (circles) derived from it, using a momentum cut-off $\Lambda = 2.0fm^{-1}$. As seen, the phase shifts from the former are well reproduced by the latter. We have also checked the half-on-shell T-matrix given by V_{NN} and by V_{low-k}, and found very good agreement between them [14]. The above agreements are expected, as we have shown that the T-matrix equivalence of Eq.(3) holds for

any Λ. In short, our numerical checks have reaffirmed that the deuteron binding energy, low energy phase shifts and low momentum half-on-shell T-matrix of V_{NN} are all preserved by V_{low-k}. As far as those physical quantities are concerned, V_{low-k} and V_{NN} are equivalent.

Having proven the "physical equivalence" of V_{low-k} and V_{NN} in the sense of the RG, we turn now to microscopic shell model calculations in which we use V_{low-k} as the input interaction. A folded-diagram formulation [13,2,3] is employed. An important feature here is that this formalism allows us to calculate the energy differences of neighboring many body systems. For example, we can calculate the energy difference of ^{18}O and the ground state energy of ^{16}O, starting from the experimental ^{17}O single particle energies (s.p.e.) and a shell model effective interaction V_{eff} derived microscopically from an underlying NN potential. This folded diagram method has been rather successfully applied to many nuclei using G-matrix interactions. [2,3] There the basic input to the calculation are the matrix elements $\langle n_1 n_2 \mid G \mid n_3 n_4 \rangle$ where G is the Brueckner G-matrix and the n's are harmonic oscillator wave functions.

Since V_{low-k} is already a smooth potential, it is no longer necessary to first calculate the G-matrix. Thus in our present work, the starting basic input are just the matrix elements $\langle n_1 n_2 \mid V_{low-k} \mid n_3 n_4 \rangle$, and thereafter our calculation procedures are exactly the same as described in references [2,3]. A model space with two valence neutrons in the $(0d_{5/2}, 0d_{3/2}, 1s_{1/2})$ shell is used for ^{18}O, and one with two and three valence protons in the $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})$ shell for ^{134}Te and ^{135}I, respectively. As customary, we use s.p.e. extracted from the experimental spectra of the corresponding single-particle valence nuclei, ^{17}O and ^{133}Sb [19]. For the absolute scaling of the sets of s.p.e., the mass-excess values for ^{17}O and ^{133}Sb have been taken from Ref. [20,21]. For ^{134}Te and ^{135}I, we assume that the contribution of the Coulomb interaction between valence protons is equal to the matrix element of the Coulomb force between the states $(g_{\frac{7}{2}})^2_{J^\pi=0^+}$.

As shown in Fig. 3, our calculated low-lying J^π states of ^{18}O agree highly satisfactorily with experiments [19]. In the same figure, results of the corresponding G-matrix calculations are also shown; the V_{low-k} results are just as good or slightly better. It may be mentioned that our V_{low-k} is slightly non-hermitian. A hermitian V_{low-k} can be obtained using the Okubo transformation [7]. We have constructed such a hermitian V_{low-k} using the Suzuki-Okamoto method [22]. We have found that the shell model energy levels given by the two V_{low-k}'s are very similar, probably because our V_{low-k} is only slightly non-hermitian. In a concurrent paper [11] we have found that V_{low-k} is almost independent of the underlying V_{NN} for the values of Λ considered here. Therefore, although the CD-Bonn potential [9] is used in our present calculations, we stress that similar results will be obtained if we calculate V_{low-k} from other models such as the Paris or Argonne V-18 potentials.

It may be mentioned that the G-matrix is energy dependent and Pauli blocking dependent, while V_{low-k} is not. This is a desirable feature, indicating that V_{low-k} may be suitable also for other nuclear regions. To study this point, we have used the same V_{low-k} in a shell model calculation of ^{134}Te as mentioned earlier. It is encouraging that our results for ^{134}Te also agree well with experiments [19] as shown in Fig. 4. Again the V_{low-k} results are just as good or slightly better than the G-matrix results. We emphasize that we have used *the same* V_{low-k} interaction in both ^{18}O and ^{134}Te calculations, and it appears to work equally well for both nuclei. This is in marked contrast to the traditional approach in which one has to use different G-matrices for different mass regions, as the associated Pauli blocking operators are different. This is an appealing result, as it suggests the possibility for a common shell-model interaction that attenuates much of the dependence on the V_{NN} model and is suitable for a wide range of nuclei.

It is of interest to investigate if the same V_{low-k} is suitable for nuclei with more than two valence nucleons. It is primarily for this purpose we have carried out the shell model calculation of ^{135}I mentioned earlier, using the same V_{low-k}. Our results are shown in Fig. 5. It is gratifying that the calculated excitation spectra are in very good agreement with experiments [19]. We note that the valence interaction energy for the three valence nucleons given by our calculation is slightly overbound, by about 0.3 MeV. This may be an indication of the need of a weak three-body force for this nucleus, which has three valence nucleons . (Our V_{low-k} is a two-body interaction.) We plan to study this topic in a future work.

An important issue is what value one should use for Λ. Guided by general EFT arguments, the minimum value for Λ must be large enough so that V_{low-k} explicitly contains the necessary degrees of freedom for the physical system. For example, 2π exchanges are important for low energy nuclear physics, and to adequately include the corresponding degree of freedom we need to have Λ_{min} larger than $\sim m_{2\pi}, i.e. \sim 1.4 fm^{-1}$. In fact we have found that V_{low-k} varies strongly with Λ when it is smaller than that value. A general signal for Λ_{min} is when the calculated physical quantities first become insensitive to Λ [5]. Conversely, we want Λ to be smaller than the short distance scale Λ_{max} at which the model dependence of the different V_{NN} starts to creep in [5]. Systems in which these two constraints are consistent with each other (i.e., $\Lambda_{min} < \Lambda_{max}$) are amenable to EFT-RG inspired effective theories, as they possess a clear separation of scales between the relevant long wavelength modes and the model dependent short distance structure. We have found [11] that Λ_{max} should not be much greater than 2.0-2.5 fm^{-1} as this is the scale after which V_{low-k} first becomes strongly dependent on the particular V_{NN} used. There is another consideration: Most NN potentials are constructed to fit empirical phase shifts up to $E_{lab} \approx 350$ MeV [9]. Since $E_{lab} \leq 2\hbar^2\Lambda^2/M$, M being the nucleon

mass, and one should require V_{low-k} to reproduce the same empirical phase shifts, a choice of Λ in the vicinity of 2 fm^{-1} would seem to be appropriate .

Guided by the above considerations, we have used in our calculations two values for the momentum cut-off, namely $\Lambda = 2.0$ and 2.2 fm^{-1} as shown in Figs. 3 to 5. It is satisfying to see that the results are rather insensitive to the choice of Λ, in harmony with the EFT philosophy mentioned earlier. Perhaps more importantly, both are in satisfactory agreement with experiments.

In summary, we have investigated a RG-EFT inspired approach to shell model calculations that is a "first step" towards a model independent calculation that uses one common interaction over a wide range of nuclei. Using the KLR folded diagram approach in conjunction with the ALS iteration method, we have performed a RG decimation where the model dependent pieces of V_{NN} models are integrated out to obtain a nearly unique low momentum potential V_{low-k}. This V_{low-k} preserves the deuteron pole as well as the low energy phase shifts and half-on-shell T matrix. We have used V_{low-k}, which is a smooth potential, directly in shell model calculations of ^{18}O, ^{134}Te and ^{135}I without first calculating the G matrix. The results are all in satisfactory agreement with experiment, and they are insensitive to Λ in the neighborhood of $\Lambda \approx 2\ fm^{-1}$. We do feel that V_{low-k} may become a promising and reliable effective interaction for shell model calculations of few valence nucleons, over a wide range of nuclear regions.

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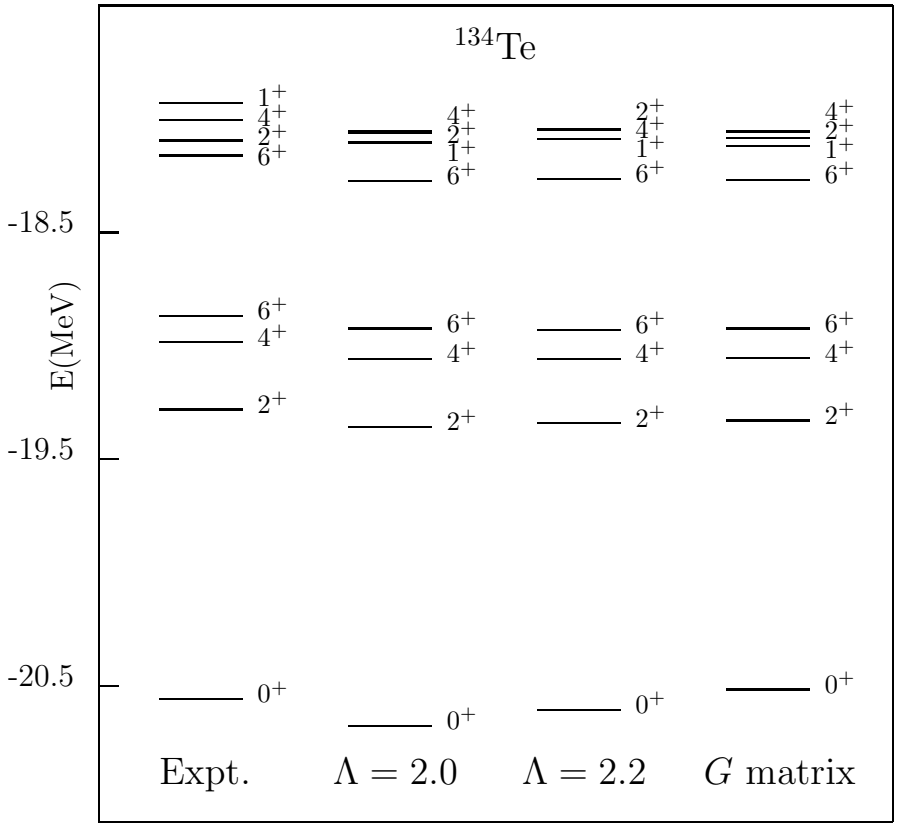
FIG. 1. Folded-diagram factorization of the half-on-shell T-matrix.

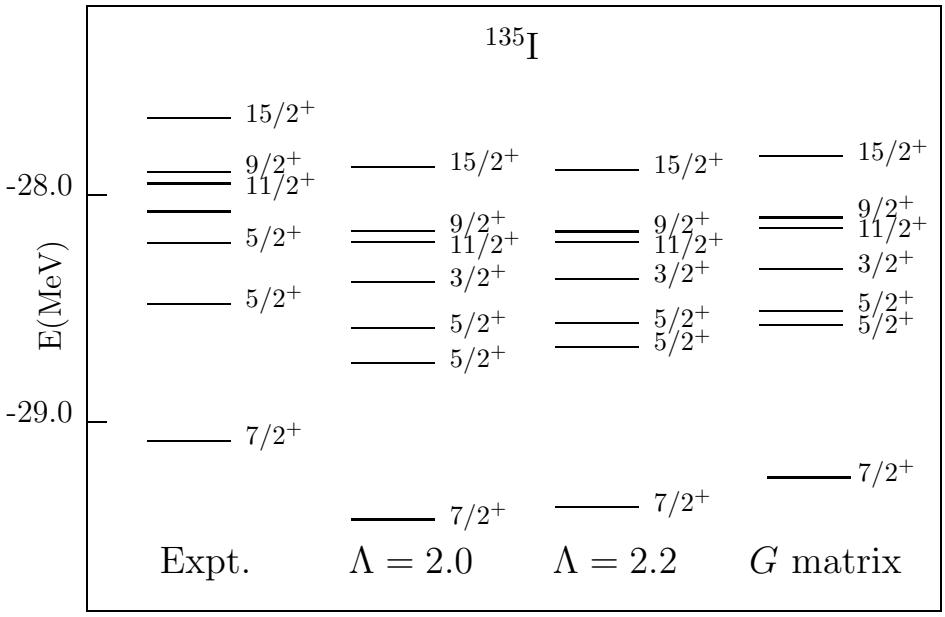
FIG. 2. Comparison of phase shifts given by V_{low-k} and V_{NN}.

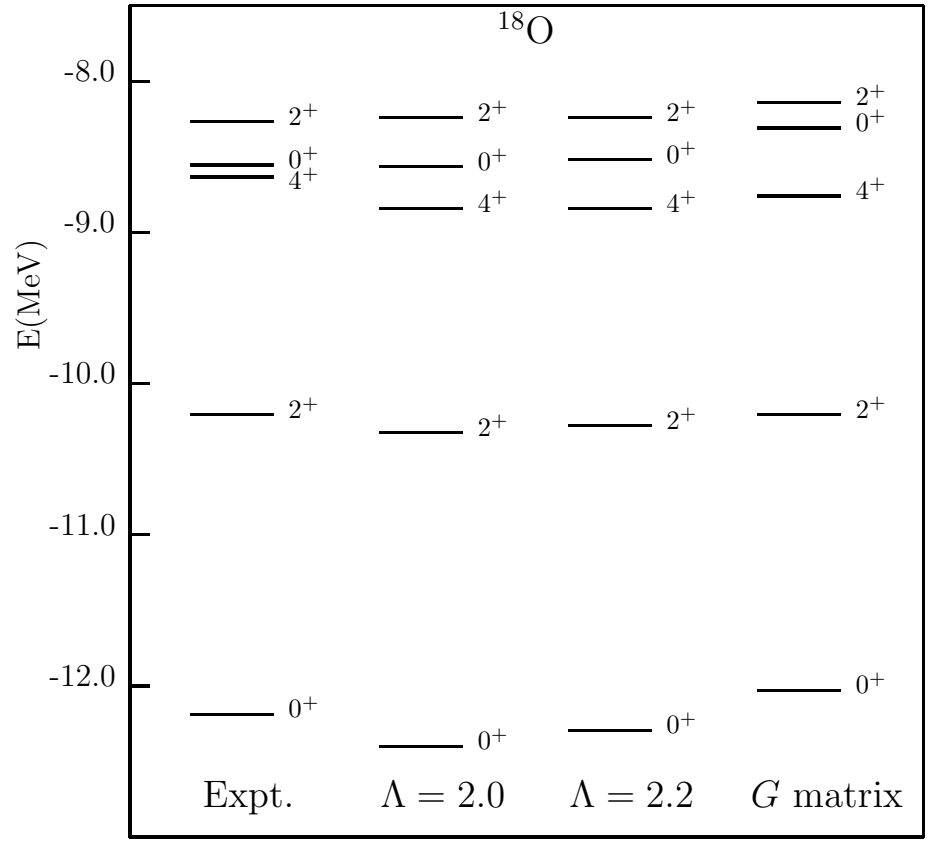
FIG. 3. Low-lying states of ^{18}O.

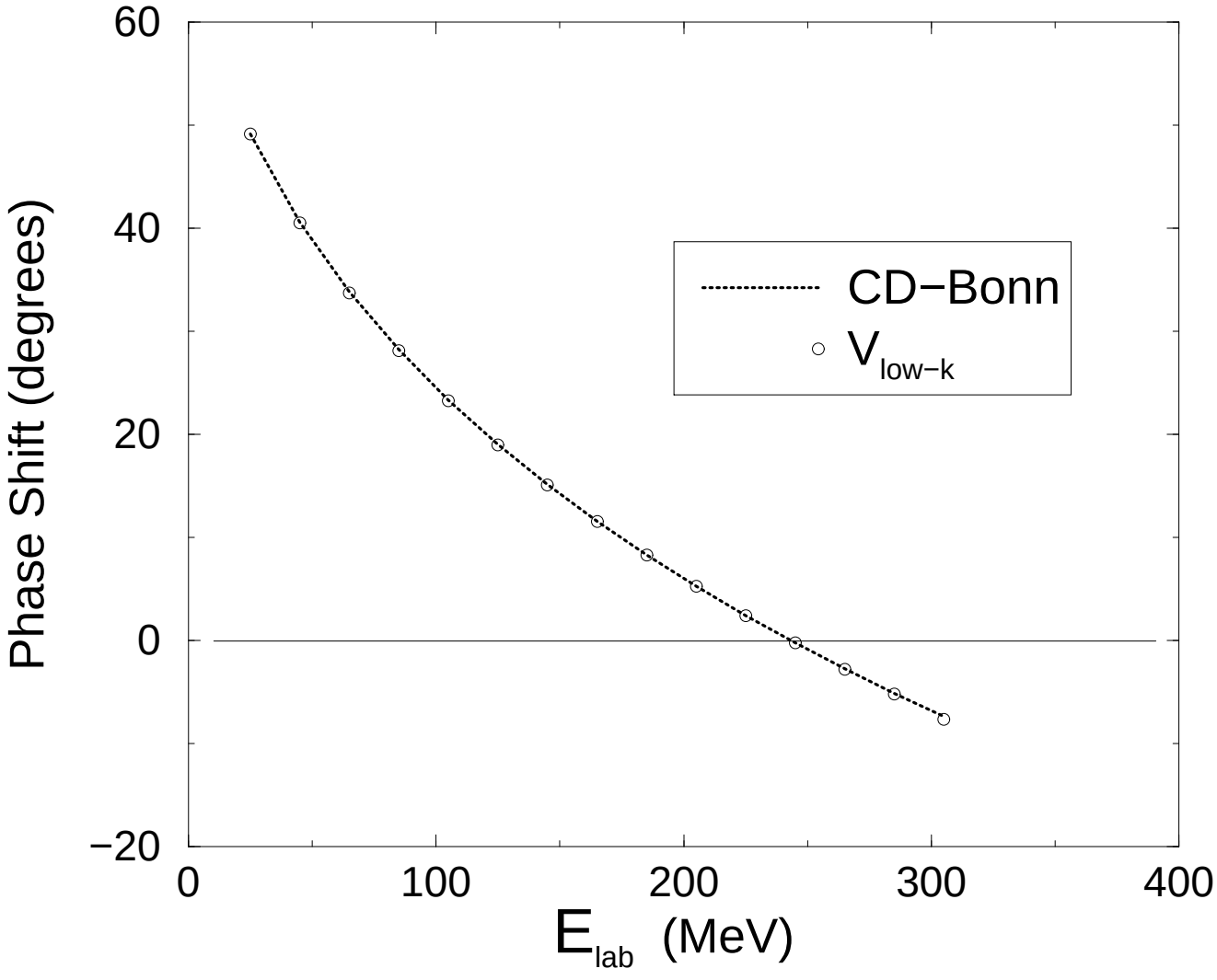
FIG. 4. Low-lying states of ^{134}Te.

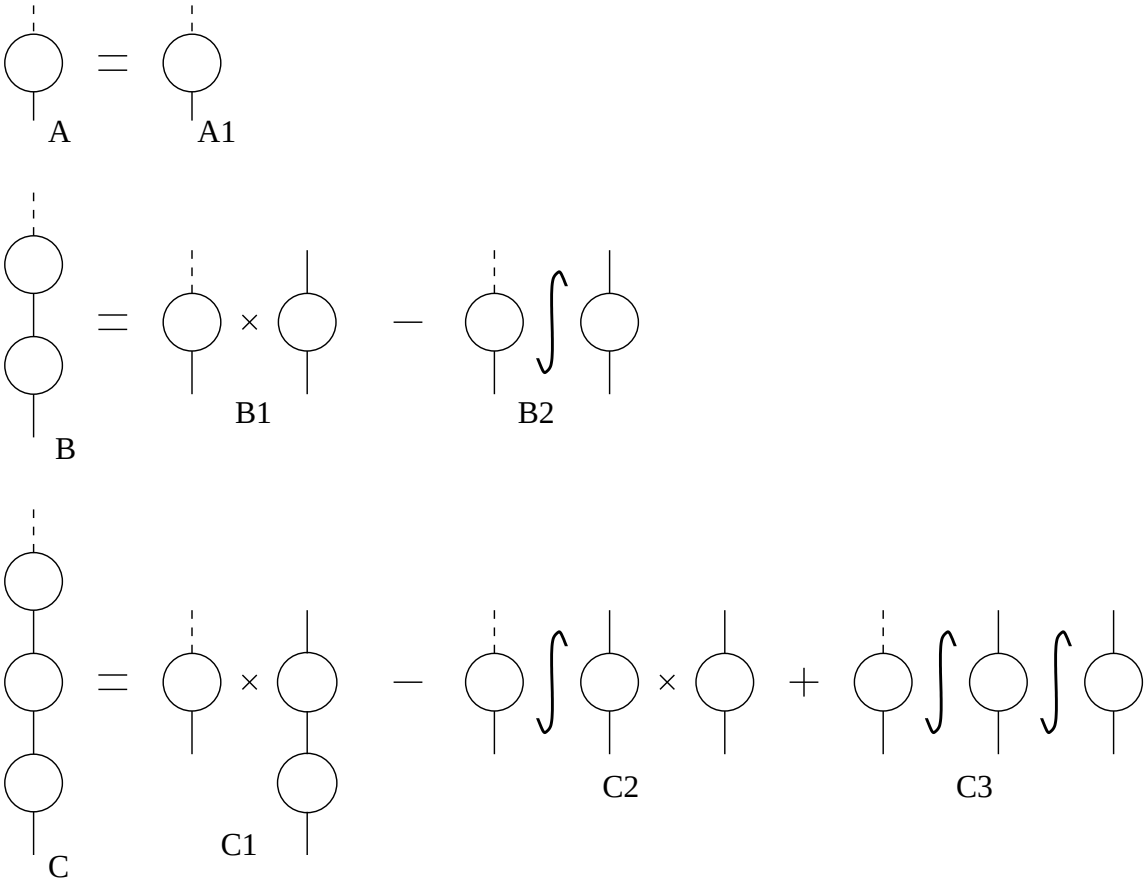
FIG. 5. Low-lying states of ^{135}I.


^{134}Te
E(MeV)
-18.5
-19.5
-20.5
1^+
4^+
2^+
6^+
4^+
2^+
1^+
6^+
2^+
4^+
1^+
6^+
4^+
2^+
1^+
6^+
6^+
4^+
2^+
0^+
6^+
4^+
2^+
0^+
6^+
4^+
2^+
0^+
6^+
4^+
2^+
0^+
Expt.
$\Lambda = 2.0$
$\Lambda = 2.2$
G matrix


^{135}I
E(MeV)
-28.0
-29.0
15/2+
9/2+
11/2+
5/2+
5/2+
7/2+
15/2+
9/2+
11/2+
3/2+
5/2+
5/2+
7/2+
15/2+
9/2+
11/2+
3/2+
5/2+
5/2+
7/2+
15/2+
9/2+
11/2+
3/2+
5/2+
5/2+
7/2+
Expt.
Λ = 2.0
Λ = 2.2
G matrix


^{18}O
-8.0
-9.0
-10.0
-11.0
-12.0
E(MeV)
2^+
0^+
4^+
2^+
0^+
Expt.
2^+
0^+
4^+
2^+
0^+
$\Lambda = 2.0$
2^+
0^+
4^+
2^+
0^+
$\Lambda = 2.2$
2^+
0^+
4^+
2^+
0^+
G matrix


60
40
20
0
−20
0
100
200
300
400
Phase Shift (degrees)
E_lab (MeV)
CD−Bonn
V_low−k


A
A1
B
B1
B2
C
C1
C2
C3